Algebras Of Pseudodifferential Operators

This book gathers peer-reviewed contributions representing modern trends in the theory of generalized functions and pseudo-differential operators. It is dedicated to Professor Michael Oberguggenberger (Innsbruck University, Austria) in honour of his 60th birthday. The topics covered were suggested by the ISAAC Group in Generalized Functions (GF) and the ISAAC Group in Pseudo-Differential Operators (IGPDO), which met at the 9th ISAAC congress in Krakow, Poland in August 2013. Topics include Columbeau algebras, ultra-distributions, partial differential equations, micro-local analysis, harmonic analysis, global analysis, geometry, quantization, mathematical physics, and time-frequency analysis. Featuring both essays and research articles, the book will be of great interest to graduate students and researchers working in analysis, PDE and mathematical physics, while also offering a valuable complement to the volumes on this topic previously published in the OT series.

This book explores various properties of quasimodular forms, especially their connections with Jacobi-like forms and automorphic pseudodifferential operators. The material that is essential to the subject is presented in sufficient detail, including necessary background on pseudodifferential operators, Lie algebras, etc., to make it accessible also to non-specialists. The book also covers a sufficiently broad range of illustrations of how the main themes of the book have occurred in various parts of mathematics to make it attractive to a wider audience. The book is intended for researchers and graduate students in number theory.

This volume consists of twenty peer-reviewed papers from the special session on pseudodifferential operators and the special session on generalized functions and asymptotics at the Eighth Congress of ISAAC held at the Peoples’ Friendship University of Russia in Moscow on August 22-27, 2011. The category of papers on pseudo-differential operators contains such topics as elliptic operators assigned to diffeomorphisms of smooth manifolds, analysis on singular manifolds with edges, heat kernels and Green functions of sub-Laplacians on the Heisenberg group and Lie groups with more complexities than but closely related to the Heisenberg group, Lp-boundedness of pseudo-differential operators on the torus, and pseudo-differential operators related to time-frequency analysis. The second group of papers contains various classes of distributions and algebras of generalized functions with applications in linear and nonlinear differential equations, initial value problems and boundary value problems, stochastic and Malliavin-type differential equations. This second group of papers are related to the third collection of papers via the setting of Colombeau-type spaces and algebras in which microlocal analysis is developed by means of techniques in asymptotics. The volume contains the synergies of the three areas treated and is a useful complement to volumes 155, 164, 172, 189, 205 and 213 published in the same series in, respectively, 2004, 2006, 2007, 2009, 2010 and 2011.

This research monograph deals with analysis on manifolds with singularities. More precisely, it presents pseudodifferential operators near edges and corners. In particular, it considers parameter-dependent edge operators and edge operators of Mellin type. The investigation of such operator families is necessary to construct operator algebras on manifolds with higher singularities. A self-contained exposition in Mellin techniques and pseudodifferential operators with operator-valued symbols is given. The algebra of parameter-dependent edge operators is constructed. Finally, Mellin operators near corner singularities are investigated. The focus is on elliptic theory. Elliptic operators on manifolds with edges are constructed as well as parametrices to elliptic elements. The equivalence of ellipticity and the Fredholm property is shown. Close to corner singularities, edge operators of Mellin type are defined, a pseudodifferential calculus is presented and parametrices are constructed. Asymptotics are treated using analytic functions and the concept of continuous asymptotic types. A technique used in the theory of partial differential equations with applications to quantum
mechanics.
This volume is based on lectures given at the workshop on pseudo-differential operators held at the Fields Institute from December 11, 2006 to December 15, 2006. The two main themes of the workshop and hence this volume are partial differential equations and time-frequency analysis. The contents of this volume consist of five mini-courses for graduate students and post-docs, and fifteen papers on related topics. Of particular interest in this volume are the mathematical underpinnings, applications and ramifications of the relatively new Stockwell transform, which is a hybrid of the Gabor transform and the wavelet transform. The twenty papers in this volume reflect modern trends in the development of pseudo-differential operators.
These notes are based on the lectures given on partial differential equations at the University of Michigan during the winter semester of 1972, with some extensions. The students to whom these lectures were addressed were assumed to have knowledge of elementary functional analysis, the Fourier transform, distribution theory, and Sobolev spaces, and such tools are used without comment. In this monography, we develop one tool, the calculus of pseudo differential operators, and apply it to several of the main problems of partial differential equations.
For the past 25 years the theory of pseudodifferential operators has played an important role in many exciting and deep investigations into linear PDE. Over the past decade, this tool has also begun to yield interesting results in nonlinear PDE. This book is devoted to a summary and reconsideration of some used of pseudodifferential operator techniques in nonlinear PDE. The book should be of interest to graduate students, instructors, and researchers interested in partial differential equations, nonlinear analysis in classical mathematical physics and differential geometry, and in harmonic analysis.
The aim of the book is to present new results in operator theory and its applications. In particular, the book is devoted to operators with automorphic symbols, applications of the methods of modern operator theory and differential geometry to some problems of theory of elasticity, quantum mechanics, hyperbolic systems of partial differential equations with multiple characteristics, Laplace-Beltrami operators on manifolds with singular points. Moreover, the book comprises new results in the theory of Wiener-Hopf operators with oscillating symbols, large hermitian Toeplitz band matrices, commutative algebras of Toeplitz operators, and discusses a number of other topics.
By generalizing the notion of the degree of a map from the sphere into the unitary group we define higher index (or degree) and eta invariants for the algebra of pseudodifferential operators obtained by p-fold suspension; the index arises in case p is even and the eta invariant in case p is odd. These functionals have similar properties to the usual index and the generalized eta functional, for the once suspended case, discussed earlier by the first author, except that the higher eta invariants are not multiplicative. For p even the index distinguishes components of the open set of elliptic elements and, for p odd, the eta invariant, by virtue of the locality of its variation, defines higher 'divisor flows' (generalizing the spectral flow) which give the obstruction for an elliptic element to have an invertible perturbation by regularizing operators. Both these functionals are shown to arise as pairings in the Hochschild-de-Rham or cyclic homology of the appropriate algebras.
This book develops three related tools that are useful in the analysis of partial differential equations (PDEs), arising from the classical study of singular integral operators: pseudodifferential operators, paradifferential operators, and layer potentials. A theme running throughout the work is the treatment of PDE in the presence of relatively little regularity. The first chapter studies classes of pseudodifferential operators whose symbols have a limited degree of regularity; the second chapter shows how paradifferential operators yield sharp estimates on the action of various nonlinear operators on function spaces. The third chapter applies this material to an assortment of results in PDE, including regularity results for elliptic PDE with rough coefficients, planar fluid flows on rough domains, estimates on Riemannian manifolds given weak bounds on Ricci tensor, div-curl estimates, and results on propagation of singularities for wave equations with rough coefficients. The last chapter studies the method of layer potentials on Lipschitz domains, concentrating on applications to boundary problems for elliptic PDE with variable coefficients.

The ISAAC Group in Pseudo-Differential Operators (IGPDO) met at the Fifth ISAAC Congress held at Università di Catania in Italy in July, 2005. This volume consists of papers based on lectures given at the special session on pseudodifferential operators and invited papers that bear on the themes of IGPDO. Nineteen peer-reviewed papers represent modern trends in pseudo-differential operators. Diverse topics related to pseudo-differential operators are covered.

Algebras of Pseudodifferential Operators

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Authored by well-known researchers, this book presents its material as accessible surveys, giving readers access to comprehensive coverage of results scattered throughout the literature. A unique source of information for graduate students and researchers in mathematics and theoretical physics, and engineers interested in the subject.

This volume consists of the plenary lectures and invited talks in the special session on pseudodifferential operators given at the Fourth Congress of the International Society for Analysis, Applications and Computation (ISAAC) held at York University in Toronto, August 11-16, 2003. The theme is to look at pseudo-differential operators in a very general sense and to report recent advances in a broad spectrum of topics, such as pde, quantization, filters and localization operators, modulation spaces, and numerical experiments in wavelet transforms and orthonormal wavelet bases.

The present thesis is concerned with certain aspects of differential and pseudodifferential operators on infinite dimensional spaces. We aim to generalize classical operator theoretical concepts of pseudodifferential operators on finite dimensional spaces to the infinite dimensional case. At first we summarize some facts about the canonical Gaussian measures on infinite dimensional Hilbert space riggings. Considering the naturally unitary group actions in $\mathcal{L}^2(H_\gamma, \gamma)$ given by weighted shifts and multiplication with $e^{iS_kp(t)(cdot)_0}$ we obtain an unitary equivalence $F$ between them. In this sense $F$ can be considered as an abstract Fourier transform. We show that $F$ coincides with the Fourier-Wiener transform. Using the Fourier-Wiener transform we define pseudodifferential operators in Weyl- and Kohn-Nirenberg form on our Hilbert space rigging. In the case of this Gaussian measure $\gamma$
we discuss several possible Laplacians, at first the Ornstein-Uhlenbeck operator and then pseudo-differential operators with negative definite symbol. In the second case, these operators are generators of $\mathcal{L}^{1,2}_{\gamma}$-sub-Markovian semi-groups and $\mathcal{L}^{1,2}_{\gamma}$-Dirichlet-forms. In 1992 Gramsch, Ueberberg and Wagner described a construction of generalized Hörmander classes by commutator methods. Following this concept and the classical finite dimensional description of $\Psi_{\{\rho, \delta\}}^0$ $(0 \leq \delta \leq \rho \leq 1$, $\delta$

One service mathematics has rendered the ‘Et moi ..., si j’avait su comment en revenir, human race. It has put common sense back je n’y serais point alle.’ where it belongs, on the topmost shelf next Jules Verne to the dusty canister labelled ‘discarded non sense’. The series is divergent; therefore we may be Eric 1’. Bell able to do something with it. O. Heaviside Mathematics is a tool for thought. A highly necessary tool in a world where both feedback and non linearities abound. Similarly, all kinds of parts of mathematics serve as tools for other parts and for other sciences. Applying a simple rewriting rule to the quote on the right above one finds such statements as: ‘One service topology has rendered mathematical physics .. .’; ‘One service logic has rendered com puter science .. .’; ‘One service category theory has rendered mathematics .. .’. All arguably true. And all statements obtainable this way form part of the raison d’etre of this series.

This monograph is devoted to the development of the theory of pseudo-differential operators on spaces with symmetries. Such spaces are the Euclidean space $\mathbb{R}^n$, the $n$-torus $T^n$, compact Lie groups and compact homogeneous spaces. The book consists of several parts. One of our aims has been not only to present new results on pseudo-differential operators but also to show parallels between differential approaches to pseudo-differential operators on different spaces. Moreover, we tried to present the material in a self-contained way to make it accessible for readers approaching the material for the first time. However, different spaces on which we develop the theory of pseudo-differential operators require different backgrounds. Thus, while operators on the euclidean space in Chapter 2 rely on the well-known Euclidean Fourier analysis, pseudo-differential operators on the $n$-torus and more general Lie groups in Chapters 4 and 10 require certain backgrounds in discrete analysis and in the representation theory of compact Lie groups, which we therefore present in Chapter 3 and in Part III, respectively. Moreover, anyone who wishes to work with pseudo-differential operators on Lie groups will certainly benefit from a good grasp of certain aspects of representation theory. That is why we present the main elements of this theory in Part III, thus eliminating the necessity for the reader to consult other sources for most of the time. Similarly, the backgrounds for the theory of pseudo-differential operators on $S^1$ and $SU(2)$ developed in Chapter 12 can be found in Chapter 11 presented in a self-contained way suitable for immediate use.

Pseudo-differential operators were initiated by Kohn, Nirenberg and Hörmander in the sixties of the last century. Beside applications in the general theory of partial differential equations, they have their roots also in the study of quantization first envisaged by Hermann Weyl thirty years earlier. Thanks to the understanding of the connections of wavelets with other branches of mathematical analysis, quantum physics and engineering, such operators have been used under different names as mathematical models in signal analysis since the last decade of the last century. The volume investigates the mathematics of quantization and signals in the context of pseudo-differential operators, Weyl transforms, Daubechies operators, Wick quantization and time-frequency localization operators. Applications to quantization, signal analysis and the modern theory of PDE are highlighted.

This volume, like its predecessors, is based on the special session on pseudo-differential operators, one of the many special sessions at the 11th ISAAC Congress, held at Linnaeus University in Sweden on August 14-18, 2017. It
includes research papers presented at the session and invited papers by experts in fields that involve pseudo-differential operators. The first four chapters focus on the functional analysis of pseudo-differential operators on a spectrum of settings from Z to R^n to compact groups. Chapters 5 and 6 discuss operators on Lie groups and manifolds with edge, while the following two chapters cover topics related to probabilities. The final chapters then address topics in differential equations.

The $0$-calculus on a manifold with boundary is a micro-localization of the Lie algebra of vector fields that vanish at the boundary. It has been used by Mazzeo, Melrose to study the Laplacian of a conformally compact metric. We give a complete characterization of those $0$-pseudodifferential operators that are Fredholm between appropriate weighted Sobolev spaces, and describe $\mathcal{C}^\ast$-algebras that are generated by $0$-pseudodifferential operators. An important step is understanding the so-called reduced normal operator, or, almost equivalently, the infinite dimensional irreducible representations of $0$-pseudodifferential operators. Since the $0$-calculus itself is not closed under holomorphic functional calculus, we construct submultiplicative Frechet $\Psi^\ast$-algebras that contain and share many properties with the $0$-calculus, and are stable under holomorphic functional calculus ($\Psi^\ast$-algebras in the sense of Gramsch). There are relations to elliptic boundary value problems.

The analysis of differential equations in domains and on manifolds with singularities belongs to the main streams of recent developments in applied and pure mathematics. The applications and concrete models from engineering and physics are often classical but the modern structure calculus was only possible since the achievements of pseudo-differential operators. This led to deep connections with index theory, topology and mathematical physics. The present book is devoted to elliptic partial differential equations in the framework of pseudo-differential operators. The first chapter contains the Mellin pseudo-differential calculus on R^+ and the functional analysis of weighted Sobolev spaces with discrete and continuous asymptotics. Chapter 2 is devoted to the analogous theory on manifolds with conical singularities, Chapter 3 to manifolds with edges. Employed are pseudo-differential operators along edges with cone-operator-valued symbols.

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